loop which decays at t_{rec}: $M_{cu} \simeq 4\times10^{-4} M_{galaxy}$. Objects with $M < M_{cu}$ are seeded by loops which decay at $t_d(R) \simeq t_{rec}$. $t_d(R)$ is given by $t_d(R) = (\gamma G\mu)^{-1}R$, with $\gamma \simeq 5$. The mass accreted by such a loop can be determined from (4) by expansion in $R - (\gamma G\mu)t_{rec}$. We find that in the limit $M \to 0$, $n(M) \sim M^{-1/2}$. For clusters $(M > M_J)$, we have the same $n(M)$ as with CDM, which has been shown to fit the data rather well.[15]

Our model does not explain the exponential decay of the luminosity function for galaxies at the bright end without invocation of the Rees-Ostriker stability arguments. The distribution of dark baryons is also an open question. Dark baryons could explain the halos of dwarf spheroidals and could also effect the ratio of mass to luminous mass. These issues deserve further attention.

We conclude that the cosmic-string theory with HDM is an interesting cosmological model which deserves further study. There are testable differences compared with a model with CDM. Flat halo-rotation curves, a characteristic mass function, and smaller galaxy masses are the main predictions. Similar conclusions have also recently been reached by Bertschinger and Watts.[16] Neutrino clustering in a more general context has been considered by Cowsik and co-workers.[17]

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Natural Quantum State of Matter Fields in Quantum Cosmology

Sumio Wada[(a)]
Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB39EW, United Kingdom
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The initial-value problem for scale factor of the universe $a = 0$ for quantum states (wave functions) of free scalar fields in the de Sitter metric $a \propto \cosh(ht)$ with imaginary t is studied. It turns out that almost all the initial conditions give the same quantum state. Some effects of the quantization of the metric are also discussed.

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In the quantum field theories in flat space-time, the Minkowski vacuum has a privileged status because of Poincaré invariance. In curved space-time, however, there is no analogous privileged quantum state. Even a vacuum is not uniquely defined.

The purpose of this work is to reexamine this situation in the context of quantum cosmology.[1,2] Quantum cosmology (cosmology with quantum gravity and quantum matter fields) can reproduce quantum field theories in classical curved space-time after some approximations,[3] but it also brings about some new aspects. Most importantly, the concept of background space-time should be extended to a complex one. In some models the lapse function (or, equivalently, the time) becomes imaginary when a (the scale factor of the Robertson-Walker universe) becomes small. Then Schrödinger equations become diffusion equations, and some interesting features emerge, especially because the Hamiltonians are singular at $a = 0$. By use of the de Sitter model with scalar fields, its implications on the initial-value problem (at $a = 0$) of wave functions (quantum states in the Schrödinger picture) are studied. It is found that a natural quantum state emerges.

In the first part of this Letter this model is treated as if the space-time is classical (though complex). In the second part the de Sitter model is quantized through the Wheeler–De Witt equation and quantum effects are studied.

Consider the de Sitter metric, $dt^2 - a^2(t)\, d\Omega_3^2$ $[a(t) = h^{-1}\cosh(ht)]$, where $d\Omega_3^2$ is the metric of the unit S^3, and h^2 is $\frac{1}{3}$ of the cosmological constant. The units $\frac{2}{3}\pi m_P^2 = 1$ will be used. First quantum theories of free scalar fields in this classical space-time are discussed, not only in the region $a > h^{-1}$ but all the way down to $a = 0$ in which t is imaginary.

The Hamiltonian of a scalar field ϕ in the above metric is diagonalized by decomposition in terms of S^3 spherical harmonics $\mathcal{Y}_{nlm}$ as $\phi(t,x) = \sum \mathcal{Y}_{nlm}(x)\phi_{nlm}(t)$. In the following we will keep the wave number n, suppressing the other two, and write ϕ_n instead of ϕ_{nlm}. The Schrödinger picture is used, in which a state (a wave function) Ψ is a function of t and ϕ_n, and satisfies the Schrödinger equation

$$\left[i\hbar\, \partial/\partial t - \sum_n H_n \right] \Psi = 0, \tag{1}$$

where the Hamiltonian H_n is

$$H_n = (1/2a^3)(-\hbar^2 \partial^2/\partial\phi_n^2 + \omega_n^2 \phi_n^2),$$

with $\omega_n^2 = (n^2-1)a^4 + (m^2 + 12h^2\xi)a^6$ (m and ξ are the mass and the coupling of ϕ to the scalar curvature, respectively).

First let us study solutions of (1) of the form

$$\Psi = C(t) \prod_n \exp(-A_n \phi_n^2 / 2\hbar).$$

From the $O(\phi_n^2)$ part of (1) we get

$$\pm a^3 (1 - h^2 a^2)^{1/2} dA_n/da + A_n^2 - \omega_n^2 = 0. \tag{2}$$

The positive and the negative signs correspond to the positive imaginary time and the negative imaginary time, respectively. By writing $A_n \equiv \pm a^3 v_n^{-1} dv_n/da$ we get

$$(1 - h^2 a^2)\left(\frac{d^2}{da^2} v_n + \frac{3}{a}\frac{d}{da} v_n \right) + \frac{\omega_n^2}{a^6} - v_n = 0. \tag{3}$$

This is analytically solvable, but I am interested only in the region $a \approx 0$, where the solution becomes simply $v_n \simeq a^{n-1} + \xi_n a^{-n-1}$ (ξ_n is an integration constant). This means that

$$A_n \simeq \pm a^2 \frac{(n-1) - \xi_n (n+1) a^{-2n}}{1 + \xi_n a^{-2n}}. \tag{4}$$

Its behavior is very different in $t = i|t|$ (the positive sign) from in $t = -i|t|$ (the negative sign). The former is discussed first. The main observation is that the solution $A_n \approx a^2(n-1)$ (i.e., $\xi_n = 0$) has a special status in the following sense: Let us consider the initial-value problem at $a = a_0$ $(\to 0)$. The trajectory which passes $A_n = A_n^{(0)}$ at $a = a_0$ is given by

$$\xi_n = \frac{(n-1)a_0^2 - A_n^{(0)}}{A_n^{(0)} + (n+1)a_0^2} a_0^{2n}. \tag{5}$$

Then take the limit $a_0 \to 0$. We find that $\xi_n = 0$ whatever the value of $A_n^{(0)}$ is, unless $A_n^{(0)} + (n+1)a_0^2$

$\to O(a^{2n+2})$. We conclude that almost all the initial conditions at $a=0$ give the same quantum state, $\xi_n=0$.

To further convince the reader, I derive the same result from a different point of view. The evolution of a wave function is given by the path-integral form

$$\Psi(\phi_n,t)=\int d\phi_n' K(\phi_n,t;\phi_n',t')\Psi(\phi_n',t'),$$
$$K=\int \mathcal{D}\phi_n \exp\left[i\int_{t'}^{t}\frac{\mathcal{L}}{\hbar}\right]\propto\exp\left[\frac{iS(\phi_n^c)}{\hbar}\right]. \tag{6}$$

The last expression means that $\phi_n^{(t)}$-dependent factor of K is given by the action S of the classical solution ϕ_n^c which connects ϕ_n at t and ϕ_n' at t'. It turns out that the equation of motion for ϕ_n^c is nothing but (3) with $v_n\leftrightarrow\phi_n^c$, and it is already known that two independent solutions, $\phi_n^{(1)}(t)$ and $\phi_n^{(2)}(t)$, can be chosen so that $\phi_n^{(1)}\to 0$ and $\phi_n^{(2)}\to\infty$ as $a\to 0$. Let us write the solution ϕ_n^c which connects ϕ_n at t and ϕ_n' at t' as $\phi_n^c=A\phi_n^{(1)}+B\phi_n^{(2)}$. Then, in the limit $a(t')\to 0$ with ϕ_n' finite, we get $B\to 0$ with A finite.

Now I calculate the action $S(\phi_n^c)$ of this trajectory. Because of the equation of motion, it is given by the boundary values as

$$S(\phi_n^c)=\left[\frac{a^3}{2}\phi_n^c\frac{d}{dt}\phi_n^c\right]_{t'\to i\pi/2h}^{t}=\frac{a^3}{2}\frac{d\phi_n^{(1)}/dt}{\phi_n^{(1)}}\bigg|_{t}\phi_n^2,$$

where I used the facts that $B\to 0$ and $\phi_n^c(t)=\phi_n$. This is independent of ϕ_n'! From (6), therefore, I conclude that Ψ at t is independent of its initial form at $t'\to i\pi/2h$ $[a(t')=0]$. In particular, when $a(t)$ itself is small (but finite), $\phi_n^{(1)}\propto a^{n-1}$ and we get $\Phi\propto\exp\{-a^2(n-1)\phi_n^2/2\hbar\}$, which coincides with the previous conclusion $\xi_n=0$.

This argument is more general than the previous one in the sense that I did not assume any functional form of $\Psi(a(t')\to 0)$. However, I assumed that ϕ_n' is finite, which means that the contribution from $\phi_n'\to\infty$ in the integral (6) should be negligible. When we take an initial condition $\Psi\propto\exp\{-A(t')\phi_n'^2/2\hbar\}$ with $A(a(t')\to 0)\to 0$, we need more careful analysis. Suppose that both $a(t)$ $(\equiv a)$ and $a(t')$ $(\equiv a')$ are small but finite. Since we know the behavior of $\phi_n^{(1)}$ and $\phi_n^{(2)}$, we can calculate $S(\phi_n^c)$ and, therefore, the integration in (6) explicitly. The result is

$$\Psi(t)\propto\exp\left\{-\frac{1}{2\hbar}\left[(n-1)a^2-\frac{4n^2a^2a'^2}{A(t')+(n+1)a'^2}\left(\frac{a'}{a}\right)^{2n}\right]\phi_n^2\right\},$$

from which it is easy to see that $\Psi(t)$ is independent of $A(a(t')\to 0)$ unless

$$A(a(t'))+(n+1)a'^2\to O(a'^{2n+2}).$$

This also agrees with the previous conclusion.

I explain what kind of state has been picked up from the above two arguments. The state Ψ is a vacuum annihilated by the operator $-i\partial/\partial\phi_n-A_n\phi_n$. The relation between the annihilation operator and the original field ϕ_n tells us[3] that the negative-frequency model u_n associated with this vacuum is given by $(a^3/u_n)du_n/dt=iA_n$, which means that $u_n\to v_n$ as $a\to 0$ (because $da/dt\to i$). Our vacuum ($\xi_n=0$) is such that $u_n\to v_n\to 0$ as $a\to 0$. This is nothing but the Bunch-Davies vacuum (the Euclidean vacuum).[4] However, this is not necessarily true when we quantize the metric as will be shown below.

All the arguments after (4) are on the case in which we go to $a=0$ along the positive imaginary time. The same arguments apply to the other case, in which the time is negative imaginary, but the result is different. The same argument as just below (4) tells us that a natural quantum state is the one with $\xi_n=0$, but here this means $A_n\approx-(n-1)a^2$ (<0). The argument based on the path integral gives the same result because $S(\phi_n^c)$ changes the sign. Because A_n is negative, this state is not acceptable, at least in free-field theories. If we define u_n by $(a^3/u_n)du_n/dt=iA_n$ again, it turns out that u_n becomes a (not negative- but) positive-frequency mode when continued to the real time.

A difference between the two cases was pointed out before from another point of view.[3,5] In Ref. 3 it was studied how the acceptable wave functions ($A_n>0$) in the region $a>h^{-1}$ evolve when a goes to zero. When t is positive imaginary, A_n remains positive only when it corresponds to the Bunch-Davies vacuum. All the A_n's of other vacua, which are related to this through the Bogoliubov transformation, become negative as a becomes small [and eventually converge to $-(n+1)a^2$ as $a\to 0$]. This apparently troublesome feature has become a nice one from the viewpoint of the initial-value problem. Because all the other vacua converge to $A_n\approx-(n+1)a^2$, it has become possible to pick up the Bunch-Davies vacuum as a unique natural choice. When t becomes negative imaginary, on the other hand, A_n's for all the vacua acceptable in the region $a>h^{-1}$ converge to $(n-1)a^2$ as $a\to 0$; this might sound good because A_n does not become negative. From the viewpoint of the initial-value problem, however, this means that we cannot pick up a unique acceptable vacuum at $a=0$.

Now we have found that, though a quite generic class of initial conditions at $a=0$ gives a natural quantum state, the result depends on whether we use $t\to i|t|$ or $t\to -i|t|$. Which is a correct way to study the initial-value problem? I argue that this should be answered in the context of quantum gravity, i.e., by quantizing the background metric. There is another reason why we should quantize the metric. Quantum field theories in classical background space-time, which have been discussed so far, are derived from quantum gravity from the $\hbar$ expansion. But $a=0$ is the turning point of the potential for a and the $\hbar$ expansion is not good there.

In the following I study (some parts of) effects of quantum gravity, quantizing the scale factor a through the Wheeler-De Witt equation. The equation for the present de Sitter model is written as

$$\left[H_0+\sum H_n\right]\Psi(a,\phi_n)=0,\quad H_0=\hbar^2\left[\frac{1}{2a}\partial_a^2+\frac{\alpha}{a}\partial_a+\frac{\beta}{a^3}\right]-\tfrac{1}{2}(a-h^2a^3). \tag{7}$$

H_n was shown in (1). H_0 comes from Einstein's action for a, $-a(da/dt)^2/2+(a-\lambda a^3)/2$, and α and β are arbitrary constants which represent ambiguities in the operator ordering and other possible quantum corrections.[6]

Suppose that the wave function has a form $\Psi=\Psi_0(a)\prod_n\exp\{-A_n(a)\phi_n^2/2\hbar\}$. Then from the $O(\phi_n^0)$ part and the $O(\phi_n^2)$ part of (7), we get

$$\hbar^2(\partial_a^2\Psi_0/2a\Psi_0+\alpha\partial_a\Psi_0/a^2\Psi_0+\beta/a^3)-(a-h^2a^3)/2-\hbar\sum A_n/2a^3=0, \tag{8}$$
$$(\hbar/2a)(\partial_a\Psi_0/\Psi_0+\alpha/a)\partial_aA_n+(\hbar/4a)\partial_a^2A_n-(1/2a^3)(-A_n^2+\omega_n^2)=0. \tag{9}$$

I ignore higher-order terms $O(\phi_n^4)$. There is not much meaning in them when the free Hamiltonian is used for ϕ_n. First note that (9) corresponds to (2). If the second term of (9) is ignored (which I will assume for a while), then (2) and (9) take the identical form by introduction of the time to the latter through

$$-i\hbar(\partial_a\Psi_0/\Psi_0+\alpha/a)=-a\,da/dt. \tag{10}$$

However, the relation between a and t becomes the previous one $a=h^{-1}\cosh(ht)$ only in the leading order of the $\hbar$ expansion. In the leading order, I write $\Psi_0\approx\exp(iS_0/\hbar)$ and (8) becomes $-(\partial_aS_0)^2-a^2(1-\lambda a^2)\approx 0$. This together with (10) implies $\pm i(1-h^2a^2)^{1/2}=da/dt$, which gives the required result. However $a=0$ is the turning point of the potential $(a-h^2a^3)/2$ and Ψ_0 should be determined more carefully.

If the last term of (8) (back reactions) can be ignored in the limit $a\to 0$ [which will be justified later by (12)], (8) means that $\Psi_0\propto a^{k-\alpha}$ where $2k=1-\{(1-2\alpha)^2-8\beta\}^{1/2}$. Then (9) without the second term becomes

$$\hbar ka\,\partial_aA_n+A_n^2-\omega_n^2=0. \tag{11}$$

This is an analog of (2). We can solve this equation by writing $A_n=\hbar kav_n^{-1}dv_n/da$. (11) becomes a linear differential equation and we get $v_n=v_n^{(1)}+\xi_nv_n^{(2)}$, where ξ_n is an integration constant and $v_n^{(1)}=1+(n^2-1)a^4/16\hbar^2k^2+\cdots$ and $v_n^{(2)}=\ln a+O(a^4)$. This means that

$$A_n\simeq\frac{(n^2-1)a^4/4\hbar k+\xi_n\hbar k}{1+\xi_n\ln a}. \tag{12}$$

At this stage I consider effects of the second term of (9) which have been ignored. (12) means that $A_n\simeq\hbar k/\ln a$ (if $\xi_n\neq 0$) or $(n^2-1)a^4/4\hbar k$ (if $\xi_n=0$) at $a\to 0$. When the second term is included, the coefficients have to change to $\hbar(k-\frac{1}{2})/\ln a$ and $(n^2-1)a^4/4\hbar(k+\frac{4}{3})$, respectively. Therefore, A_n at $a\to 0$ is better expressed by

$$A_n\simeq\frac{(n^2-1)a^4/4\hbar(k+\frac{4}{3})+\xi_n\hbar(k-\frac{1}{2})}{1+\xi_n\ln a}.$$

(The argument here is admittedly not rigorous.)

Now let us consider the initial-value problem again. The trajectory which passes $A_n=A_n^{(0)}$ at $a=a_0$ is given by

$$\xi_n=\frac{(n^2-1)a^4/4\hbar(k+\frac{4}{3})-A_n^{(0)}}{A_n^{(0)}\ln a-\hbar(k-\frac{1}{2})}.$$

We find again that $\xi_n=0$ in the limit $a=0$ whatever the value of $A_n^{(0)}$ is, unless $A_n^{(0)}\to\hbar(k-\frac{1}{2})/\ln a$ as $a\to 0$. The solution with $\xi_n=0$ has a special status in the same sense as before.

Now implications of the above calculation are discussed. In general, a solution Ψ_0 of (8) can be written as $\Psi_0=A\Psi_0^{(1)}+B\Psi_0^{(2)}$ where $\Psi_0^{(1)}\propto a^{k-\alpha}$ and $\Psi_0^{(2)}\propto a^{1-\alpha-k}$. In the above I implicitly assumed that $A\neq 0$ (and k is real). Then $[\partial_a\Psi_0/\Psi_0]_{a\to 0}$ is uniquely determined and the natural state becomes the one with $A_n(a\to 0)\simeq\hbar a^4/(k+\frac{4}{3})$ (whose sign depends on the value of k). However, the evolution of this state depends on the value of A/B. For larger a (but $<h^{-1}$) where the WKB approximation becomes valid, Ψ_0 can be expressed as a linear combination of $\exp(|S_0|/\hbar)$ and $\exp(-|S_0|/\hbar)$ (S_0 is the action of the de Sitter metric at imaginary time). If the value of A/B is such that the former dominates, then we find that t [determined from (10)] goes down on (or parallel to) the imaginary axis as a increases. Then we get (2) with the positive sign. If the value of A/B is such that $\exp(-|S_0|/\hbar)$ dominates, then t goes up on (parallel to) the imaginary axis as a increases.[3] Then we get (2) with the negative sign. If neither dominates, the result is more complicated. Numerical calculation is needed to get the exact evolution of A_n from $a=0$, but A_n, if it is positive near $a=0$, is likely to remain positive when $\exp(|S_0|/\hbar)$ dominates. [We can see this by considering the flow of the solutions in the $a-A_n$ plane, which is given by (5) or by an exact expression for $\xi=m=0$ in (4.3) of Ref. 3]. In short, the ambiguities in the determination of a natural quantum state of matter fields can be solved by the choice of the

metric part $\Psi_0(a)$ of the wave function. There are some speculations about it,[7] but I will not go into details here.

I add three comments below. Firstly, the above analysis of the Wheeler-De Witt equation does not depend on the detailed form of the potential for a $(a-\hbar^2 a^3$ in the above example). The behavior of Ψ_0 is governed by α and β. Therefore this analysis is applicable to other models such as the de Sitter metric with the flat spatial section, to which my first argument (semiclassical gravity) is not applicable because $a=0$ can be reached at real t.

Secondly, (11) means that my calculation can be interpreted as that of quantum fields in classical space-time $a=a(t)$ which is determined by (10). This space-time deviates from the de Sitter metric as a goes to zero. [In fact, $a^3=\hbar k(t_0-it)$ where t_0 is an integration constant.] As a result the natural quantum state that was picked up above deviates from the Bunch-Davies vacuum. The negative-frequency mode u_n can be calculated as before. It turns out that $u_n(a\to 0)$ is nonvanishing but finite when $\xi_n=0$. Note also that $u_n(a\to 0)\to\infty$ for the vacua with $\xi_n\neq 0$, which was also the case in (5).

Finally I take note that the natural quantum state which was picked up in my first argument has been claimed to be natural from some other arguments before. It was proposed by Halliwell and Hawking[8] that the initial condition at $a=0$ in the path integral should be such that matter field configuration ϕ is regular in (Euclidean) space-time, which means that $\phi_n=0$ at $a=0$ from the single valuedness of ϕ. This gives my natural state.[3] (In fact, I showed here that "ϕ_n=finite" is enough.) The same state can also be picked up by the demand that the action of matter fields which is calculated from $a=0$ is finite.[5] This is related to the fact that only this state has a negative-frequency mode which does not diverge in the limit $a\to 0$. Ratra[9] picked up the same state by demanding that it should coincide with the ground state of the time-independent harmonic oscillator in the limit $a\to 0$.

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(a)Permanent address: Institute of Physics, University of Tokyo-Komaba, Komaba 3-8-1, Tokyo 153, Japan.

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Comment on "Spinning Cosmic Strings and Quantization of Energy"

Mazur[1] has recently derived the intriguing result that in the background space-time of a spinning string, energy must *necessarily* be quantized. It is well known[2] that such a result obtains in the background space-time of a gravitational monopole: Time must be a periodic coordinate and energy must quantized. By use of an exact analogy between rotation in general relativity and magnetic fields, both of these results can be translated to the more familiar context of electromagnetism. The latter corresponds to Dirac's celebrated result that electric charge is quantized in the field of a magnetic monopole. However, the electromagnetic analog of the former is clearly not true. The spinning string corresponds, in electromagnetism, to a charged infinite solenoid or flux tube. The presence of a solenoid does not imply charge quantization. We therefore thought it worthwhile to examine more closely the arguments that led to the main conclusion of Ref. 1, and we find that energy *need not* be quantized in the background of a spinning string. This restores the complete parallel between the electromagnetic and gravitational cases.

The fallacy in the argument is elementary and becomes apparent if one correctly keeps track of the identification of points under multivalued coordinate transformations. Following Ref. 1, the metric of a spinning string is

$$ds^2=-(dt-A\,d\phi)^2+\rho^2\alpha^2 d\phi^2+d\rho^2+dz^2, \tag{1}$$

where $\alpha=1-4GM$, $A=-4GJ$, and M and J are two parameters representing the mass and angular momentum per unit length of the string. The ranges of the coordinates (t,ϕ,ρ,z) are $-\infty<t<\infty$, $0\le\phi\le 2\pi$, $0<\rho<\infty$, $-\infty<z<\infty$, and the points

$$(t,\phi,\rho,z) \text{ and } (t,\phi+2\pi,\rho,z) \tag{2}$$

are identified. The transformation $T=t-A\phi$, $\phi'=\alpha\phi$ reduces the line element (1) to locally Minkowskian form. In terms of the new coordinates (T,ϕ',ρ,z), (2) translates into the identification of (T,ϕ',ρ,z) and $(T-2\pi A,\phi+2\pi\alpha,\rho,z)$. Implementing correctly the single valuedness of the wave function, we find

$$\Psi(T-2\pi A,\phi'+2\pi\alpha,\rho,z)=\Psi(T,\phi',\rho,z), \tag{3}$$

and not

$$\Psi(T-2\pi A,\phi',\rho,z)=\Psi(T,\phi',\rho,z), \tag{4}$$

as implied in Eq. (7) of Ref. 1. We now consider eigenstates of "energy" and "angular momentum,"

$$\Psi(T,\phi',\rho,z)=\exp\left[-\frac{i}{\hbar}(E'T-L'\phi')\right]f(\rho,z),$$

and find using (3) that E' and L' obey the condition

$$AE'+\alpha L'=\text{integer}, \tag{5}$$

which does *not* imply the quantization of E'. If in the original coordinate system, the variables conjugate to t and ϕ are denoted E and L, respectively, then $E'=E$, $\alpha L'=L-AE$, and we see that Eq. (5) simply represents the quantization condition for the angular momentum L.

One is, of course, at liberty to identify points

$$(t,\phi,\rho,z) \text{ and } (t+P,\phi,\rho,z) \tag{6a}$$

in the spinning-string space-time and arrive at a new space-time in which energy *is* quantized in units h/P. But this is not forced on us (unlike in the case of the monopole). If one does make the identification (6a) with the value

$$P=2\pi A \tag{6b}$$

suggested by Mazur, one finds that a *globally* well-defined coordinate transformation $T=t-A\phi$ reduces the (Kerr-type) spinning-string metric to

$$ds^2=-dT^2+\alpha^2\rho^2 d\phi^2+d\rho^2+dz^2,$$

a (Schwarzschild-type) nonspinning metric. Thus the condition (6) renders the angular momentum of the string unobservable. Conversely, requiring that the angular momentum of the string be unobservable leads to (6) and energy quantization. But this is an *extra* assumption that one need not feel obliged to make.

The question of causality-violating regions is peripheral to the whole discussion. One cannot avoid such regions by declaring time to be a periodic coordinate. In fact, this leads to causality violation over all space-time and not just for $\rho<\rho_0$.

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J. Samuel and B. R. Iyer
Raman Research Institute
Bangalore 560080, India

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